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APJ Abdul Kalam Technological University
Thiruvananthapuram

Abstract

APJAKTU - Academic - Minor in Mathematics - basket of 5 courses - approved - Orders issued.

ACADEMIC SECTION

U.O.No. 1531/2020/KTU Thiruvananthapuram, Dated: 06.11.2020

Read:-Minutes of the 10th Academic Council meeting held on 27/10/2020 in item no.010:3:02.

ORDER

Curriculum Committee of Mathematics has recommended a basket of 5 courses of Minor in Mathematics that are to be offered from semesters three to seven of B.Tech 2019 scheme. Vide paper cited the 10th Academic Council resolved to approve the basket of 5 courses and curriculum and syllabi of courses to be followed in semesters 3 & 4 of B.Tech Programme.

Sanction has been accorded by the Vice Chancellor to incorporate the basket of 5 courses related to Minor in Mathematics as envisaged in clause R.11. 6 of B.Tech 2019 Regulations.

The Basket contains the following courses:

Sl.No.	Course Code	Course Name	Semester of Study
1	MAT281	Advanced Linear Algebra	S3
2	MAT282	Mathematical Optimization	S4
3	MAT381	Random Process and Queuing Theory	S5
4	MAT382	Algebra and Number Theory	S6
5	MAT481	Functional Analysis	S7

The Curriculum and Syllabi of the two courses for 3rd and 4th semesters B.Tech are also attached hereunder.

Orders are issued accordingly.

Sd/-

Dr. Bijukumar R *
Dean (Academic) in Charge

Copy to:-

1. JD(IT) for publishing in the portal & website.
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Section Officer

* This is a computer system (Digital File) generated letter. Hence there is no need for a physical signature.

010.4.01 MINOR IN MATHEMATICS

There are requests from many corners to have one or more baskets for MINOR in MATHEMATICS. The CC of Mathematics is pleased to offer one basket as listed below along with the syllabus for S3 and S4.

SEMESTER	BASKET I			
	COURSE NO.	COURSE NAME	HOURS	CREDIT
S3	MAT281	Advanced Linear Algebra	4	4
S4	MAT 282	Mathematical Optimization	4	4
S5	MAT 381	Random Process and Queuing Theory	4	4
S6	MAT 382	Algebra and Number theory	4	4
S7	MAT 481	Functional Analysis	4	4

CODE MAT	Advanced Linear Algebra	CATEGORY	L	T	P	CREDIT
		B. Tech Minor (S3)	**3**	**1**	**0**	**4**

Preamble: This course introduces the concept of a vector space which is a unifying abstract frame work for studying linear operations involving diverse mathematical objects such as n-tuples, polynomials, matrices and functions. Students learn to operate within a vector and between vector spaces using the concepts of basis and linear transformations. The concept of inner product enables them to do approximations and orthogonal projects and with them solve various mathematical problems more efficiently.

Prerequisite: A basic course in matrix algebra**.**

Course Outcomes: After the completion of the course the student will be able to

CO 1	Identify many of familiar systems as vector spaces and operate with them using vector space tools such as basis and dimension.
CO 2	Understand linear transformations and manipulate them using their matrix representations.
CO 3	Understand the concept of real and complex innerproduct spaces and their applications in constructing approximations and orthogonal projections
CO 4	Compute eigen values and eigen vectors and use them to diagonalize matrices and simplify representation of linear transformations
CO 5	Apply the tools of vector spaces to decompose complex matrices into simpler components, find least square approximations, solution of systems of differential equations etc.

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	3	3	3	2	1			1	2		2
CO 2	3	3	3	3	2	1			1	2		2
CO 3	3	3	3	3	2	1			1	2		2
CO 4	3	2	3	2	1	1			1	2		2
CO 5	3	3	3	3	2	1			1	2		2

Assessment Pattern

Bloom's Category	Continuous Assessment Tests		End Semester Examination
	1	2	
Remember	5	5	10
Understand	10	10	20
Apply	10	10	20
Analyse	10	10	20
Evaluate	15	15	30
Create			

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Show that the $S_1 = \{(x, y, 0) \in R^3\}$ is a subspace of R^3 and $S_2 = \{(x, y, z) \in R^3 : x + y + z = 2\}$ is not a subspace of R^3

2. Let S_1 and S_2 be two subspaces of a finite dimensional vector space. Prove that $S_1 \cap S_2$ is also a subspace. Is $S_1 \cup S_2$ s subspace. Justify your answer.

3. Prove that the vectors $\{(1,1,2,4), (2,-1,5,2), (1,-1,-4,0), (2,1,1,6)\}$ are linearly independent

4. Find the null space of $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{bmatrix}$ and verify the rank nullity theorem for $m \times n$matrix in case of A

Course Outcome 2 (CO2)

1. Show that the transformation $T; R^2 \to R^3$defined by $T(x, y) = (x - y, x + y, y)$

is a linear transformation.

2. Determine the linear mapping $\varphi; R^2 \to R^3$ which maps the basis (1,0,0), (0,1,0) and (0,0,1)to the vectors (1,1), (2,3)and(−1,2). Hence find the image of (1,2,0)
3. Prove that the mapping $\varphi; R^3 \to R^3$ defined by $T(x, y, z) = (x + y, \quad y + z, z + x)$ is

an isomorphism

Course Outcome 3(CO3):

1. Prove that the definition $f(u, v) = x_1y_1 - 2x_1y_2 + 5x_2y_2$for $u = (x_1, y_1)$ and $v = (x_2, y_2)$ is an inner product in R^2.

2. Prove the triangle inequality $\|u + v\| \leq \|u\| + \|v\|$ in any inner product space.

3. Find an orthonormal basis corresponding to the basis $\{1, tcost, sint\}$of the subspace of the vector space of continuous functions with the inner product defined by $\int_0^\pi f(t)g(t)dt$ using Gram Schimdt process .

Course Outcome 4 (CO4):

1. Consider the transformation $T: R^2 \to R^2$ defined by $(x, y) = (x - y, 2x - y)$. Is T diagonalizable. Give reasons.

2. Use power method to find the dominant eigen value and corresponding eigen vector of $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 18 & -1 & -7 \end{bmatrix}$.

3. Prove that a square matrix A is invertible if and only if all of its eigen values are non-zero.

Course Outcome 5 (CO5):

1. Find a singular value decomposition of $\begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 2 & -2 \end{bmatrix}$

2. Find the least square solution to the system of equations
$$x + 2y + z = 1, 3x - y = 2, 2x + y - z = 2, x + 2y + 2z = 1$$

3. Solve the system of equations $2x_1 + x_2 + x_3 = 2$,$x_1 + 3x_2 + 2x_3 = 2$, and $3x_1 + x_2 + 2x_3 = 2$ by LU decomposition method.

Syllabus

Module 1

Vector Spaces, Subspaces -Definition and Examples. Linear independence of vectors, Linear span, Bases and dimension, Co-ordinate representation of vectors. Row space, Column space and null space of a matrix

Module 2

Linear transformations between vector spaces, matrix representation of linear transformation, change of basis, Properties of linear transformations, Range space and Kernel of Linear transformation, Inverse transformations, Rank Nullity theorem, isomorphism

Module 3

Inner Product: Real and complex inner product spaces, properties of inner product, length and distance, Cauchy-Schwarz inequality, Orthogonality , Orthonormal basis, Gram Schmidt orthogonalizationprocess. Orthogonal projection. Orthogonal subspaces, orthogonal compliment and direct sum representation.

Module 4

Eigen values, eigenvectors and eigen spaces of linear transformation and matrices, Properties of eigen values and eigen vectors, Diagonalization of matrices, orthogonal diagonalization of real symmetric matrices, representation of linear transformation by diagonal matrix, Power method for finding dominant eigen value,

Module 5

LU-decomposition of matrices, QR-decomposition, Singular value decomposition, Least squares solution of inconsistent linear systems, curve-fitting by least square method, solution of linear systems of differential equations by diagonalization

Text Books

1. Richard Bronson, Gabriel B. Costa, *Linear Algebra-an introduction,* 2nd edition, Academic press, 2007

2. Howard Anton, Chris Rorres, *Elementary linear algebra: Applications versio,* 9th edition, Wiley

References

1. Gilbert Strang, *Linear Algebra and It's Applications,* 4th edition, Cengage Learning, 2006
2. Seymour Lipschutz, Marc Lipson, *Schaum's outline of linear algebra*, 3rd Ed., McGrawHill Edn.2017
3. David C Lay*, Linear algebra and its applications,3rd edition, Pearson*
4. Stephen Boyd, LievenVandenberghe, I*ntroduction to Applied Linear Algebra: Vectors, Matrices,and Least Squares,* Cambridge University Press, 2018
5. W. Keith Nicholson, *Linear Algebra with applications*, 4th edition, McGraw-Hill, 2002

Assignments:

Assignment should include specific problems highlighting the applications of the methods introduced in this course in science and engineering.

Course Contents and Lecture Schedule

No	Topic	No. of Lectures
1	**Vector spaces (9 hours)**	
1.1	Defining of vector spaces , example	2
1.2	Subspaces	1
1.3	Linear dependence, Basis , dimension	3
1.4	Row space, column space, rank of a matrix	2
1.5	Co ordinate representation	1
2	**Linear Mapping (9 hours)**	
2.1	General linear transformation, Matrix of transformation.	2
2.2	Kernel and range of a linear mapping	1
2.3	Properties of linear transformations,	2
2.4	Rank Nullity theorem.	1
2.5	Change of basis .	2
2.6	Isomorphism	1
3	**Inner product spaces (9 hours)**	
3.1	Inner Product: Real and complex inner product spaces,	2
3.2	Properties of inner product, length and distance	2
3.3	Triangular inequality, Cauchy-Schwarz inequality	1
3.4	Orthogonality, Orthogonal complement, Orthonormal bases,	1
3.5	Gram Schmidt orthogonalizationprocess, orthogonal projection	2
3.6	Direct sum representation	1
4	**Eigen values and Eigen vectors (9 hours)**	
4.1	Eigen values and Eigen vectors of a linear transformation and matrix	2
4.2	Properties of Eigen values and Eigen vectors	1
4.3	Diagonalization., orthogonal diagonalization	4
4.4	Power method	1
4.5	Diagonalizable linear transformation	1

5	**Applications (9)**	
5.1	LU decomposition, QR Decomposition	2
5.2	Singular value decomposition	2
5.3	Least square solution	2
5.4	Curve fitting	1
5.5	Solving systems of differential equations.	2

CODE MAT	**Mathematical optimization**	**CATEGORY**	**L**	**T**	**P**	**CREDIT**
		B. Tech Minor (S4)	**3**	**1**	**0**	**4**

Preamble: This course introduces basic theory and methods of optimization which have applications in all branches of engineering. Linear programming problems and various methods and algorithms for solving them are covered. Also introduced in this course are transportation and assignment problems and methods of solving them using the theory of linear optimization.Network analysis is applied for planning, scheduling, controlling, monitoring and coordinating large or complex projects involving many activities. The course also includes a selection of techniques for non-linear optimization

Prerequisite: A basic course in the solution of system of equations, basic knowledge on calculus.

Course Outcomes: After the completion of the course the student will be able to

CO 1	Formulate practical optimization problems as linear programming problems and solve them using graphical or simplex method.
CO 2	Understand the concept of duality in linear programming and use it to solve suitable problems more efficiently .
CO 3	Identify transportation and assignment problems and solve them by applying the theory of linear optimization
CO 4	Solve sequencing and scheduling problems and gain proficiency in the management of complex projects involving numerous activities using appropriate techniques.
CO 5	Develop skills in identifying and classifying non-linear optimization problems and solving them using appropriate methods.

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	3	3	3	2	1			1	2		2
CO 2	3	3	3	3	2	1			1	2		2
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End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Without sketching find the vertices of the possible solutions of $-x + y \leq 1, 2x + y \leq 2\ ,\ x,\ y \geq 0$
2. Solve the LPP $Max\ \ 8x_1 + 9x_2$ subject to $2x_1 + 3x_2\ \leq 50\ ,\ 3x_1 + x_2 \leq 3\ ,x_1 + 3x_2 \leq 70\ ,\ x_1, x_2 \geq 0$ by simplex method
3. Solve the LPP $Max\ -x_1 + 3x_2$ subject to$x_1 + 2x_2\ \geq 2\ ,\ 2x_1 + 6x_2 \leq 80, x_1 \leq 4\ ,\ x_1, x_2 \geq 0$ by Big M method.

Course Outcome 2 (CO2)

1. Formulate the dual of the following problem and show that dual of the dual is the primal $Max\ 5x_1 + 6x_2$ subject to $x_1 + 9x_2\ \geq 60\ ,\ 2x_1 + 3x_2 \leq 45\ ,\ x_1, x_2 \geq 0$
2. Using duality principle solve $Min\ \ 2x_1 + 9x_2 + x_3$ subject to $x_1 + 4x_2 + 2x_3\ \geq 5\ ,\ 3x_1 + x_2 + 2x_3\ \geq 4$,, $x_1, x_2 \geq 0$
3. Use dual simplex method to solve $Min\ \ z = x_1 + 2x_2 + 4x_3$ subject to $2x_1 + 3x_2 - 5x_3\ \leq 2\ ,\ 3x_1 - x_2 + 6x_3\ \geq 1\ ,x_1 + x_2 + x_3\ \leq 3\ ,\ x_1 \geq 0 x_2 \leq 0\ ,\ x_3$ unrestricted

Course Outcome 3(CO3):

1. Explain the steps involved in finding the initial basic solution feasible solution of a transportation problem by North West Corner rule..
2. A company has factories A, B and C which supply warehouses at W_1 , W_2 and W_3. Weekly factory capacities are 200, 160 and 90 units respectively. Weekly warehouse

requirement are 180,120 and 150 respectively. Unit shipping cost in rupees is as follows

16	20	12
14	8	16
26	24	16

Determine the optimal distribution of this company to minimise the shipping cost

3. In a textile sales emporium, sales man A, B and C are available to handle W, X Y and Z. Each sales man can handle any counter . The service time in hours of each counter when manned by each sales man is as follows

	A	B	C	D
W	41	72	39	52
X	22	29	49	65
Y	27	39	60	51
Z	45	50	48	52

Course Outcome 4 (CO4):

1. Draw the network diagram to the following activities.

Activity	1-2	1-3	1-4	2-5	3-5	4-6	5-6
Time duration	2	4	3	1	6	5	7

2. The following table gives the activities in a construction project and other relevant information

Activity	1-2	1-3	1-4	2-5	3-5	4-6	5-6
Time duration	2	4	3	1	6	5	7

Find the free , total and independent float for each activity and determine the critical activities.

3. For a project given below find (i) the expected time for each activity (ii) T_E, T_L values of all events (iii) the critical path.

Task	A	B	C	D	E	F	G	H	I	J	K
Least time	4	5	8	2	4	7	8	4	3	5	6

Greatest time	6	9	12	6	10	15	16	8	7	11	12
Most likely time	5	7	10	4	7	8	12	6	5	8	9

Course Outcome 5 (CO5):

1. Consider the unconstrained optimization problem $max f(\boldsymbol{x}) = 2x_1x_2 + x_2 - {x_1}^2 - 2{x_2}^2$. Starting from the initial solution $(x_1, x_2) = (1,1)$ interactively apply gradient search procedure with $\in = 025$ to get an approximate solution.

2. Consider the following nonlinear programming problem.

$$Max\ f(\boldsymbol{x}) = \frac{1}{1+x_2} \text{subject to } x_1 - x_2 \leq 2, x_1 \geq 0, .\ x_2 \geq 0$$

   Use KKT condition to show that $(x_1, x_2) = (4, 2)$ is not an optimal solution

3. Minimize $f = -4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2$ subject to $2\,x_1 + x_2 \leq 6,\ x_1 - 4\,x_2 \leq 0, x_1 \geq 0,\ x_2 \geq 0$ using Quadratic programming method.

Syllabus

MODULE I

Linear Programming – 1 : Convex set and Linear Programming Problem – Mathematical Formulation of LPP, Basic feasible solutions, Graphical solution of LPP, Canonical form of LPP, Standard form of LPP, slack variables and Surplus variables, Simplex Method, Artificial variables in LPP, Big-M method.

MODULE II

Linear Programming – 2 : Two-phase method, Degeneracy and unbounded solutions of LPP, Duality of LPP, Solution of LPP using principle of duality, Dual Simplex Method.

MODULE III

Transportationand assignment problems: Transportation Problem, Balanced Transportation Problem, unbalanced Transportation problem. Finding basic feasible solutions – Northwest corner rule, least cost method, Vogel's approximation method. MODI method.Assignment problem, Formulation of assignment problem, Hungarian method for optimal solution, Solution of unbalanced problem. Travelling salesman problem

MODULE IV

Sequencing and Scheduling: Introduction, Problem of Sequencing, the problem of n jobs and two machines, problem of m jobs and m machines, Scheduling Project management-Critical path method (CPM), Project evaluation and review technique (PERT), Optimum scheduling by CPM, Linear programming model for CPM and PERT.

MODULE V

Non Linear Programming: Examples nonlinear programming problems- graphical illustration**.** One variable unconstrained optimization, multiple variable unconstrained optimization- gradient search**.** TheKarush –Kuhn Tucker condition for constraint optimization-convex function and concavefunction.Quadratic programming-modified simplex method-restricted entry rule, Separable programming.

TextBook

1. Frederick S Hillier, Gerald J. Lieberman, Introduction to Operations Research, Seventh Edition, McGraw-Hill Higher Education, 1967.
2. KantiSwarup, P. K. Gupta, Man Mohan, Operations Research, Sultan Chand & Sons, New Delhi, 2008.

Reference

1. Singiresu S Rao, Engineering Optimization: Theory and Practice ,New Age International Publishers, 1996
2. H A Taha, Operations research : An introduction , Macmillon Publishing company,1976
3. B. S. Goel, S. K. Mittal, Operations research, PragatiPrakashan, 1980
4. S.D Sharma, "Operation Research", KedarNath and RamNath - Meerut , 2008.
5. **Phillips, Solberg Ravindran** ,Operations Research: Principles and Practice, **Wiley,2007**

Assignments:

Assignment should include specific problems highlighting the applications of the methods introduced in this course in science and engineering.

Course Contents and Lecture Schedule

No	**Topic**	**No. of Lectures**
1	**Linear programming – I (9 hours)**	
1.1	Convex set and Linear Programming Problem – Mathematical Formulation of LPP	2
1.2	Basic feasible solutions, Graphical solution of LPP	2
1.3	Canonical form of LPP, Standard form of LPP, slack variables and Surplus variables, Artificial variables in LPP	1
1.4	Simplex Method	2
1.5	Big-M method.	2
2	**Linear programming – II (9 hours)**	
2.1	Two-phase method	2
2.2	Degeneracy and unbounded solutions of LPP	2
2.4	Duality of LPP	1
2.5	Solution of LPP using principle of duality	2

2.3	Dual Simplex Method.	2
3	**Transportation and assignment problems- (9 hours)**	
3.1	Balanced transportation problem	2
3.2	unbalanced Transportation problem	1
3.3	Finding basic feasible solutions – Northwest corner rule, least cost method	1
3.4	Vogel's approximation method. MODI method	2
3.5	Assignment problem, Formulation of assignment problem	1
3.6	Hungerian method for optimal solution, Solution of unbalanced problem. Travelling salesman problem	2
4	**Sequencing and Scheduling- (9 hours)**	
4.1	Introduction, Problem of Sequencing, the problem of n jobs and two machines	2
4.2	problem of m jobs and m machines	1
4.3	Scheduling Project management-Critical path method (CPM)	2
4.4	Project evaluation and review technique (PERT),	2
4.5	Optimum scheduling by CPM, Linear programming model for CPM and PERT.	2
5	**Non Linear Programming - (9 hours)**	
5.1	Examples , Graphical illustration, One variable unconstrained optimization	2
5.2	Multiple variable unconstraint optimization-- gradient search	2
	The Karush –Kuhn Tucker condition for constraint optimization	1
5.3	Quadratic programming-modified simplex method-	2
5.5	Separable programming	2

Action Required: The academic council may discuss and approve